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NONPERTURBATIVE QCD CORRECTIONS TO THE BJORKEN AND GROSS-LLEWELLYN-SMITH SUM RULES

S. Fajfer

Institut za fiziku, University of Sarajevo, Sarajevo, Yugoslavia

and

Fermi National Accelerator Laboratory, Batavia, IL 60510, USA

and

R.J. Oakes

Department of Physics and Astronomy, Northwestern University
Evanston, IL 60201 USA

and

Fermi National Accelerator Laboratory, Batavia, IL 60510, USA

ABSTRACT

There are nonperturbative Quantum Chromodynamic (QCD) corrections to the Bjorken and Gross-Llewellyn-Smith sum rules coming from the nonsinglet and singlet nucleon matrix elements, respectively, of the two-quark, one-gluon operators, which appear in the operator product expansion of the bilocal product of the charged weak currents. These twist-four, spin-one QCD corrections can be calculated assuming some quark confinement model to evaluate the nucleon matrix elements. We have considered several plausible models and have found the resulting corrections to be quite model dependent. However, in all of the models these nonperturbative QCD corrections are rather small compared to the usual perturbative QCD corrections calculated in leading-log approximation.

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The perturbative Quantum Chromodynamic (QCD) corrections to the various sum rules among the deep inelastic lepton scattering structure functions, or Nachtmann moments of these structure functions, have been extensively studied and used to test QCD.[1] In addition to these perturbative QCD corrections there are nonperturbative, higher twist QCD corrections. The twist-four corrections to the structure functions, particularly their second Nachtmann moments, for electron and neutrino scattering on nucleon targets have also been investigated.[1-10] Most of these calculations have included only contributions coming from four-quark operators, neglecting the two-quark, one-gluon operators, or have found their contributions to be quite small.

However, these two-quark, one-gluon operators also contribute corrections to the Bjorken sum rule for the first Nachtmann moment of the first structure function $F_1(x,Q^2)$ in neutrino scattering on nonsinglet nucleon targets, as well as the Gross-Llewellyn-Smith sum rule for the first Nachtmann moment of the third structure function $F_3(x,Q^2)$ in neutrino scattering on singlet nucleon targets. We have calculated their twist-four, spin-one contributions to both the Bjorken and Gross-Llewellyn-Smith sum rules using the operator product expansion of the product of the charged weak currents and have considered various quark confinement models for the nucleon matrix elements. The technical aspects of the calculation have been fully discussed in the previous work on higher twist effects.[1-10] While we found these effects to be very model dependent, the resulting corrections to the sum rules were quite small in all of the models.

The Bjorken sum rule for the first Nachtmann moment of $F_1\,(x,Q^2)$ for neutrino scattering on nonsinglet nucleon targets, including the perturbative and the twist-four corrections, is of the form

$$M_1^{\nu,NS}(1,Q^2) = \frac{1}{2}\left[1 - \frac{8}{3b\ln Q^2/\Lambda^2} + \frac{2g(Q^2)}{9\,Q^2}\,A_1^-\right] \tag{1}$$

The corresponding Gross-Llewellyn-Smith sum rule for neutrino scattering on singlet nucleon targets is

$$M_3^{\nu,S}(1,Q^2) = 3\left[1 - \frac{4}{b\ln Q^2/\Lambda^2} + \frac{2g(Q^2)}{27Q^2}\,A_1^+\right] \tag{2}$$

In these expressions for the first Nachtmann moments NS and S denote nonsinglet and singlet nucleon matrix elements, respectively, and

$$g^2(Q^2)/4\pi = [(b/4\pi)\ \ln\ (Q^2/\Lambda^2)]^{-1}$$

is the strong coupling constant with $b=11-2N/3$, as usual. In the nonperturbative correction terms the nucleon matrix elements $A_1^\pm$ are defined by

$$P_\mu\ A_1^\pm = \langle P\ |\ O_{1\mu}^\pm(o)|\ P\rangle \tag{3}$$

where P_μ is the nucleon momentum. The two-quark, one-gluon operators are given by

$$O_{1\mu}^\pm(x) = 2\ \overline{q}(x)\ \gamma^\alpha\ \gamma^5\ [I_-,\ I_+]_\pm\ (\lambda^a/2)\ q(x)\ \tilde{F}^a_{\mu\alpha}(x) \tag{4}$$

with $q(x)$ being an up or down quark field and $\tilde{F}^a_{\mu\alpha}(x)$ the dual gluon field. $I_\pm$ and $\lambda^a/2$ are the usual flavor and color matrices.

To evaluate the matrix elements $A_1^\pm$ we use nucleon wave functions $|\tilde{P}\rangle$, normalized to unity, which represent the confined quarks in the nucleon center of mass. In terms of these wave functions the matrix elements are simply

$$\langle P|\ O_{1\mu}^\pm(o)\ |P\rangle = 2M\ \langle\tilde{P}\ |\int d^3x\ O_{1\mu}^\pm(x)\ |\tilde{P}\rangle \tag{5}$$

In addition, in the nucleon rest frame only the $\mu = 0$ component of the operator will have a non-zero matrix element and, therefore

$$A_1^\pm = -4\ \langle\tilde{P}\ |\int d^3x\ \overline{q}(x)\ \vec{\gamma}\ \gamma^5\ [I_-,I_+]_\pm(\lambda^a/2)q(x)\cdot\vec{B}^a(x)\ |\tilde{P}\rangle \tag{6}$$

where $\vec{B}^a(x)$ is the color magnetic field.

This color magnetic field can be calculated inside the nucleon using the QCD equations of motion[11]; viz.,

$$\vec{\nabla} \times \vec{B}^a(\vec{r}) = \vec{j}^a(\vec{r}) \tag{7}$$

where

$$\begin{aligned} \vec{j}^a(\vec{r}) &= \bar{q}(\vec{r})\, \vec{\gamma}\, (\lambda^a/2)\, q(\vec{r}) \\ &= -\frac{3}{4\pi}\, \hat{r} \times \vec{\sigma}\, (\lambda^a/2)\, \mu'(r)/r^3 \end{aligned} \tag{8}$$

with $\mu'(r)$ being the quark scalar magnetization density. For nucleon models with confinement radius R the quark wave function is of the usual spherically symmetric form

$$\psi(\vec{r}) = \begin{pmatrix} f(r) \\ \vec{\sigma} \cdot \hat{r}\, g(r) \end{pmatrix} \chi \tag{9}$$

normalized so that

$$\int |\psi(\vec{r})|^2\, d^3r = 4\pi \int_0^R \{|f(r)|^2 + |g(r)|^2\}\, r^2 dr = 1 \tag{10}$$

For such models the magnetization of a quark is given by

$$\mu(R) = \int_0^R \mu'(r')\, dr' \tag{11}$$

Solving eqs. (7)-(11), the QCD color magnetic field appearing in eq. (6) can be expressed in terms of quark operators[11]:

$$\begin{aligned} \vec{B}^a(\vec{r}) &= \vec{\sigma}\, (\lambda^a/16\pi)\, [2M(r) + \mu(R)/R^3 - \mu(r)/r^3] \\ &\quad + \hat{r}\, (\vec{\sigma}\cdot\hat{r})\, (3\lambda^a/16\pi)\mu(r)/r^3 \end{aligned} \tag{12}$$

where

$$M(r) = \int_r^R \mu'(r')/r'^3\, dr' \tag{13}$$

Turning first to the spin, flavor and color parts of the nucleon matrix elements $A_1^{\pm}$ we note that the flavor combinations of u and d quarks appearing in the Bjorken and Gross-Llewellyn-Smith sum rules are, respectively,

$$(\bar{u}u - \bar{d}d)\ (\bar{u}u + \bar{d}d) \tag{14a}$$

and

$$(\bar{u}u + \bar{d}d)\ (\bar{u}u + \bar{d}d). \tag{14b}$$

Calculating these spin, flavor and color factors in the matrix elements for protons and neutrons one finds, respectively, for the nonsinglet and singlet combinations, the factors of 4/3 and 4. Including these factors, the matrix elements can be written as

$$A_1^+ = 3A_1^- = -\, g(Q^2)/\pi \int_0^R \{|f(r)|^2\ [2M(r) + \mu(R)/R^3]$$

$$-\ 1/3\ |g(r)|^2[2M(r) + \mu(R)/R^3 - 4\ \mu(r)/r^3]\}\ r^2 dr \tag{15}$$

here, in terms of the radial wave functions, we now find

$$\mu(r) = -\, i(8\pi/3)\int_0 f^*\ (r')\ g\ (r')\ r'^3\ dr' \tag{16a}$$

and

$$M(r) = -\, i(8\pi/3)\int_r^R f^*\ (r')\ g\ (r')\ dr' \tag{16b}$$

To illustrate the size of the nonperturbative corrections, as well as their model dependence, we have considered six different quark confinement models. Four of these are variations of the MIT bag model.[11,12] The other two are a relativistic potential model[13] and a harmonic oscillator shell model.[14] (For these latter two models the limit $R \to \infty$ must be taken in the above equations.) In Table I we give $A_1^+/g(Q^2)$ for each of these six models.[15] In addition, the nonperturbative QCD corrections are compared with the perturbative corrections to emphasize that they are quite small in all cases. However, although small, it is quite evident from Table I that these twist-four, spin-one corrections are very model dependent. One might have anticipated such a significant model dependence by observing that this correction depends on the product of the "large" and "small" components of the quark wave function $\Psi(\vec{r})$, as can be seen from eqs. (15) and (16). In fact, for completely nonrelativistic confinement models the overlap integrals in eq. (16) vanish, while they are rather large in models for which the confined quarks are relativistic.

In conclusion, we have calculated the twist-four, spin-one nonsinglet and singlet contributions to the first Nachtmann moments of the two-quark, one-gluon operators in the expansion of the product of the charged weak currents. The effects on both the Bjorken sum rule, for neutrino scattering on nonsinglet nucleon targets, and the Gross-Llewellyn-Smith sum rule, for neutrino scattering on singlet nucleon targets, were found to be quite model dependent; albeit, very small in all the six models considered. In no case did these nonperturbative QCD effects exceed 2.5% of the leading-log perturbative QCD corrections. In fact, for the most popular version of the MIT bag model[11] they amounted to only about 0.1%. Our results, therefore, indicate that perturbative QCD corrections should quite accurately account for any violations of the Bjorken and Gross-Llewellyn-Smith sum rules, as assumed in the analysis of Bolognese, et al.,[16] who extracted the value $\Lambda=92^{+20}_{-36}$ MeV from the data on the Gross-Llewellyn-Smith sum rule. Our calculations indicate that the small higher-twist effects can effect such a determination of Λ by, at most, about 10%.

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REFERENCES

1. See, the example, the review of A. Buras, Rev. Mod. Phys. 52 (1980) 199.
2. R.L. Jaffe and M. Soldate, Phys. Lett. 105B (1981) 467.
3. R.L. Jaffe and M. Soldate, Phys. Rev. D26 (1982) 49.
4. E.V. Shuryak and A.I. Vainstain, Phys. Lett. 105B (1981) 65.
5. E.V. Shuryak and A.I. Vainstain, Nucl. Phys. B199 (1982) 451.
6. S. Fajfer and R.J. Oakes, Phys. Lett. 131B (1983) 192.
7. S. Fajfer and R.J. Oakes, Proceedings of the XXII International Conference on High Energy Physics, Leipzig, DDR, 1984.
8. S. Fajfer and R.J. Oakes, Phys. Rev. D30 (1984) 1585.
9. P. Castorina and P.J. Mulders, Phys. Rev. D (to be published).
10. B.A. Iijima, MIT preprint, CTP 993 (unpublished).
11. T. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D12, (1975) 2060.
12. R.H. Hackman, N.G. Deshpande, D.A. Dicus and V.L. Teplitz, Phys. Rev. D18 (1978) 2537.
13. P. L. Ferreira, J.A. Helayel and N. Zagurgy, Nuovo Cimento 55A (1980) 215.
14. P. Colić, J. Trampetić and D. Tadić, Phys. Rev. D26 (1982) 2286.
15. Our results for $A_1^+/g(Q^2)$ for the MIT Bag Model II agree with ref. 10, where this case has also been considered. However, for all the models our results are opposite in sign and smaller in magnitude than the estimate given in refs. 4 and 5.
16. T. Bolognese, P. Fritze, J. Morfin, D.H. Perkins, K. Powell and W.G. Scott, Phys. Rev. Lett. 50 (1983) 224.

Model	$A_1^+/g(Q^2)(\mathrm{GeV}^2)$	$\frac{8\pi^2}{27}\,[A_1^+/g(Q^2)]/Q^2$ (%)
MIT Bag Model I (ref. 11)	-3.53×10^{-4}	0.10
MIT Bag Model II (ref. 11)	-2.23×10^{-3}	0.65
MIT Bag Model III (ref. 12)	-6.33×10^{-4}	0.19
MIT Bag Model IV (ref. 12)	-2.77×10^{-4}	0.08
Relativistic Potential Model (ref. 13)	-8.48×10^{-3}	2.48
Harmonic Oscillator Shell Model (ref. 14)	-2.63×10^{-3}	0.77

Table I: Numerical values of $A_1^+/g(Q^2)$ for various quark confinement models and the ratio $(8\pi^2/27)\,[A_1^+/g(Q^2)]/Q^2$ of nonperturbative to perturbative QCD corrections to the Gross-Llewellyn-Smith sum rule [eq. (2)], expressed in percent, for $Q^2 = 1\ \mathrm{GeV}^2$. (For the Bjorken sum rule [eq. (1)] this ratio is larger by a factor of 3/2).